Solutions to selected exercises from Chapter 6

6.1

1. No, without any points labeled or scales given it is impossible to determine if $f(x)$ is quadratic or exponential because we cannot determine how the function is changing (either a constant change in the rate of change or percentage change).

2.

x	***f(x)***	Δy	$\Delta(\Delta y)$
-2	1	1	
-1	2	3	2
0	5	5	2
1	10	7	2
2	17		

3.

x	***f(x)***	*Change factor*
-2	$\frac{1}{9}$	3
-1	$\frac{1}{3}$	3
0	1	3
1	3	3
2	9	

4. The initial value appears to be (0,1) and the values are doubling every one unit of x so a formula would be $y = 1(2)^x$.

5. (1975, 385000) (2005, 1122000)

Average rate of change: $\dfrac{1{,}122{,}000 - 385{,}000}{2005 - 1975} = 24{,}566.67 \ \dfrac{\text{immigrants}}{\text{year}}$

30-year growth factor: $\dfrac{385{,}000}{112{,}200} \approx 2.914$

Annual growth factor: $\sqrt[30]{\dfrac{385{,}000}{1{,}122{,}000}} \approx 1.036$

30-year percentage change: 291% - 100% = 191%

Annual percentage change: 103.6% - 100% = 3.6%

6 a. The 80-year growth factor would be 2,641.

b. The 80-year percentage growth rate would be 264,000% or 2641 – 1 = 2640(100%)

c. The average annual growth factor is $\sqrt[80]{2641} = 1.103$.

d. The average annual percentage change is $\begin{array}{c} 1.103 - 1 = 0.103 \\ 0.103 \cdot 100 = 10.3\% \end{array}$

e. $I(y) = 1(1.103)^y$

f. $\begin{array}{l} I(90) = 1(1.103)^{90} \\ = \$6,788.85 \end{array}$ After 90 years, the initial investment would have risen to a value of $6,788.85 with a 10.3% percentage change.

7. We assume three years ago (in year 0) there were 33,048 courses and this year (in year 3) there are 38,183. The average rate of change is $\frac{38,183 - 33,048}{3 - 0} = \frac{5,135 \text{ courses}}{3 \text{ years}} \approx 1712 \ \frac{\text{courses}}{\text{year}}$ and the annual percentage change is $\sqrt[3]{\frac{38,183}{33,048}} = \sqrt[3]{1.1553} = 1.049$ or 104.9% - 100% = 4.9%.

Using the average rate of change, in two years we can estimate that 38,183 + 1712 (2) = 41,607.

Using the annual percentage change, in two years we can estimate that $38,183(1.049)^2 = 42,017$.

8. Either a quadratic or exponential would fit the data relatively well. There is neither a common factor or a common second difference but the data is increasing at an increasing rate in terms of its overall trend.

9a. (0, 1998000) (6, 2866500). For a linear model we assume a constant rate of change $\frac{2,866,500 - 1,998,000}{2006 - 2000} = 144,750 \ \frac{\$}{\text{year}}$ and a function $L(y) = 144,750y + 1,998,000$. The slope means that each year the average professional baseball player's salary increased by $144,750 for every year

after 2000. The *y*-intercept means that in the year 0 (2000) the average professional baseball player's salary was $1,998,000.

b. For an exponential model we assume a constant percentage change

$$2{,}866{,}500 = 1{,}998{,}000(x)^6$$
$$1.434 = x^6$$
$$x = 1.06$$

and a function $E(y) = 1{,}998{,}000(1.06)^y$. The initial value represents the average professional baseball player's salary in year 0 (2000) was $1,998,000. The growth factor, 1.06, means that the rate of change increases each year by 1.06 times the previous year's increase.

10a. From 1980 to 2005 the change factor is 0.4214, from 1990 to 1995 the change factor is 0.8564, and from 2000 to 2005 the change factor is 0.8812.

b. From 1980 to 2005 the percentage change is 58% decrease, from 1990 to 1995 the percentage change is 14% decrease, and from 2000 to 2005 the percentage change is 12% decrease. From 1980 to 2005 the value of the dollar decreased by 58%, from 1990 to 1995 the value of the dollar decreased by 14%, and from 2000 to 2005 the value of the dollar decreased by 12%.

c. The average rate of change is $\dfrac{0.512-0.928}{25-5} = \dfrac{-0.416}{20} = \dfrac{-\$0.0208}{1 \text{ year}}$. The percentage change is $\sqrt[20]{\dfrac{0.512}{0.928}} = 0.9707$.

d. If the percentage change is negative consumers' purchasing power is decreasing (their money is worth less), if the percentage change is positive consumers' purchasing power is increasing (their money is worth more), and if the percentage change is zero consumers' purchasing power is unchanging.

13. Johnny Depp's earnings is $18,000,000 and the average American's salary is $30,000. Finding the change factor we get $\dfrac{30{,}000}{18{,}000{,}000} = 0.001\overline{6}$. An average American's salary is 0.167% of Johnny Depp's so when Johnny Depp buys something the impact on his budget is always 0.167% of what it would be to the average person. Therefore each item is multiplied by the change factor of 0.00167 in order to get the "Johnny Depp" price.

14. The annual percentage growth rate for Atlanta would be

$$\sqrt[6]{1.21} \approx 1.032$$
$$1.032 - 1 = 0.032(100\%) = 3.2\%$$

The annual percentage decay rate for New Orleans would be

$$1 - 0.222 = 0.778$$
$$\sqrt[6]{0.778} \approx 0.959$$
$$1 - 0.959 \approx 0.041(100\%) = 4.1\%$$

15. If $T(y) = 2300(2)^y$ is an accurate model it should generate reasonably close estimates for the actual number of transistors. The function *T(y)* gives us the following estimates *T(0) = 2300, T(1) = 4600, T(3) = 18,400, T(7) = 294,400, T(11) = 4,710,400*, etc. These are not very close to the actual data so the model *T(y)* is not very accurate and not supportive of Moore's Law.

16.

Week	***Allowance Option 1***	***Allowance Option 2***
0	0.01	1.00
1	0.02	2.00
2	0.04	3.00
3	0.08	4.00
4	0.16	5.00
5	0.32	6.00
6	0.64	7.00
7	1.28	8.00
8	2.56	9.00
	$0.01(2)^w$	$1.00 + 1.00w$
52	$\$4.5(10^{13})$	\$53

The best option for working for only 4 or 8 weeks is Option 2 but for a year it is overwhelmingly Option 1.

17a. 118 – 10*w*

b. $118(0.90)^w$

18a. For the 8" x 5" chart, 8(0.80) = 6.4" and 5(0.80) = 4".

b. Area of new: (6.4") (4") = $\frac{25.6}{40} = 0.64$. The percent reduction in area is 36%.

c. $3 = 5(0.80)^x$
$0.6 = 0.80^x$
$x = \log_{0.8}(0.6)$
$x = 2.29$
There would need to be 3 reductions.

d. $D_1 = 8(0.80)^{10}$ $\quad D_2 = 5(0.80)^{10}$
$D_1 = 0.86$ $\quad D_2 = 0.54$

The dimensions would be 0.86” x 0.54”.

e. $h = 5(0.80)^r$ and $w = 8(0.80)^r$

19. (answers vary). In an L.A. Times article the term exponential growth refers simply to a rapidly increasing relationship. The term is used in a data-less context about the fact that China and India are growing “by leaps and bounds”. The article goes on to discuss why recycling water is essential.

20. C.

21. These data are exponential because of the common factor of 0.5.

x	***y***	$\frac{y}{x}$
0	80	$\frac{40}{80} = 0.5$
1	40	$\frac{20}{40} = 0.5$
2	20	$\frac{10}{20} = 0.5$
3	10	$\frac{5}{10} = 0.5$
4	5	$\frac{2.5}{5} = 0.5$
5	2.5	

24. These data are exponential because of the common factor of 3.

x	***y***	$\frac{y}{x}$
0	-5	$\frac{-15}{-5} = 3$
1	-15	$\frac{-45}{-15} = 3$

2	-45	$\frac{-135}{-45}=3$
3	-135	$\frac{-405}{-135}=3$
4	-405	$\frac{-1215}{-405}=3$
5	-1215	

25. a. $81.77(1.087)=88.88$.

b. If we let R represent the room rate and y be the number of years since 2005 we get $R(y)=81.77(1.087)^y$.

c. For only a relatively short period of time because if the rooms rates continued to rise at 8.7% then the cost would become too expensive for people to afford.

26. The daily decay factor is $\sqrt[8]{0.5}\approx 0.917$. If we let A_o represent the original amount of iodine we can model this situation is $I(d)=A_o(0.917)^d$. We can find out how much is left after a month by evaluating

$$\begin{aligned} I(30) &= A_o(0.917)^{30} \\ &= A_o(0.0743) \end{aligned}$$

. This means that about $\frac{7}{100}$ will be left after 30 days.

28 a. 2
b. 4
c. 8
d. 16
e. $A=2^g$
f. Year 1776 is 232 years ago. $\frac{232}{20}=11.6$ generations so
$A=2^{11.6}=3104$ ancestors.

Year 1642 is 366 years ago. $\frac{366}{20}=18.3$ generations so
$A=2^{18.3}=322,737$ ancestors.

Year 1000 is 1008 years ago. $\frac{1008}{20}=50.4$ generations so
$A=2^{50.4}=1,485,633,834,000,000$ ancestors.

31. We need to find out how many 100 feet Denver is above sea level so $\frac{5280}{100} = 52.8$. If atmospheric pressure decreases exponentially at a rate of 0.45 for every 100 feet we can model this by $\begin{aligned} A(h) &= c(1.004)^{52.8} \\ &\approx c(1.235) \end{aligned}$.

Therefore, the air pressure is reduced by 23.5%.

34. The initial value is 1 person and then the rumor spreads by doubling so $n(x) = 1(2)^x$.

35. If the chain letter is sent to 5 people by each person, y = total number of people in the chain, and x = the number of mailers since you received it.

x	***y***
0	5
1	25
2	125
3	625
4	3,125
5	15,625
6	78,125
7	390,625
8	1,953,125
9	9,765,625
10	48,828,125
11	244,140,625

36. a. On the 14^{th} day the pool will be covered with algae.

# of days neglected	***Square feet of algae***
1	$\frac{1}{16}$
2	$\frac{1}{8}$
3	$\frac{1}{4}$
4	$\frac{1}{2}$
5	1
6	2
7	4
8	8

9	16
10	32
11	64
12	128
13	256
14	**512**

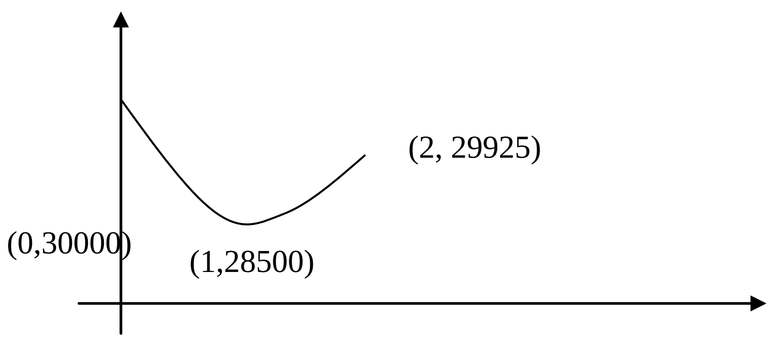
b. One day. Since the algae is doubling daily then on the 13th day it's ½ covered so by day 14 it will be completely covered.

37. III.

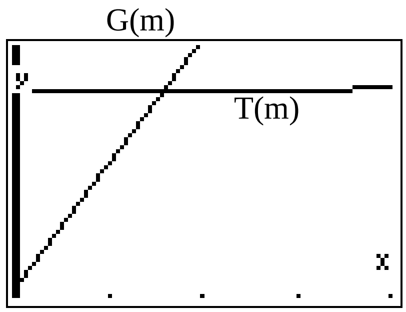
38. c. The beanstalk was 5 cm tall initially and then grew at 15% per day so the growth factor is 1.15.

39. If one's salary is $30,000 and decreased by 5% the salary would be 0.95(30,000) = $28,500. If they were then given a 5% increase the following year they would have 1.05(28,500) = 29,925 so no they would not be back to their original salary.

Year	***Salary***
0	30,000
1	*0.95(30,000) = 28,500*
2	*1.05 (28,500) = 29,925*



40 a. Gilbert's monthly growth rate is linear and can be modeled by $G(m) = 161{,}059 + 1000m$ while Tempe's monthly growth rate is exponential (because of the common monthly growth factor of $\sqrt[12]{1.00019} \approx 1.000015832$) and can be modeled by $T(m) = 162{,}652(1.000015832)^m$. The population of Gilbert will overtake Tempe's because Tempe's is so low. We can see this on both the graph and table below.



X	Y_1	Y_2
0	161059	162652
1	162059	162655
2	163059	162657
3	164059	162660
4	165059	162662
5	166059	162665
6	167059	162667

X=0

b. The growth rate of Tempe is almost equal to 1 which means no growth at all.

6.2

1. If there is a constant change factor.

2. No. Given only two points it is impossible to determine if there is a constant change factor.

3. Doubling, tripling, etc. Half-life, percentage change

4. The amount of time required for half the initial amount of a substance to remain.

5. The amount of time for something that is growing to double.

6. Letting P = the U.S. population and t = years since 2005, we can write the exponential model $P(t) = 298,213(1.9)^t$

7. $C(y) = 1,323,345,000(1.007)^y$; 1,379,907,275

8. An exponential function is not appropriate because there is a constant rate of change (0.27 quadrillion BTU's per year) meaning it is linear.

13. An exponential function is not appropriate because there is a constant rate of change ($1800 per year) meaning it is linear.

14. It would not be appropriate to model the situation because over the 40 years the percentage of adults with only an elementary education had a decay factor of 0.466 and then over the next 40 years the decay factor was only 0.2162.

19. $d(t) = 40.63(1.098)^t$ In 1995 the estimated brand name drug price was $40.63. The growth factor of 1.098 tells us that each subsequent year the brand name drug price will be 1.098 times the prior year.

20. $p(t) = 49.39(0.97)^t$. The percentage of highway accidents resulting in injuries is decreasing by 3% per year.

23. $w(t) = 2.59(1.11)^t$. It would be helpful to bottled water companies to know how the consumption of bottled water is changing.

24. $b(t) = 161.43(1.169)^t$ Using the function b(t) we estimate that the average NBA salary will be $b(30) = 161.43(1.169)^{30} = \$17,475,096$ so the model projects that the average NBA salary will be much more than $3 million in 2010.

31. It appears that Graph D when considering the graphs for problems 31 – 35. An estimate for the population for 2005 using Graph D would be 20,250,000 people.

32. D. Approximately 19,400,000

36. The function is decreasing (the change factor is less than 1), concave up, the vertical intercept is (0,21) (the initial value tells us this), and the horizontal asymptote will be the line y = 0.

37. Decreasing, concave down, vertical intercept (0,-4), horizontal asymptote is y = 0.

42. If a function has a constant percentage change it will change at a variable rate and therefore cannot be linear (a constant rate of change).

44. Disagree. For positive x values f(x) > g(x) but when you consider negative x-values f(x) will become less than g(x).

6.4

1. $7^3 = 343$
2. $5^{-6} = \dfrac{1}{15,625}$
3. $10^{2.778} \approx 600$
4. $e^y = x$

5. $\ln 24.53 \approx 3.2$

6. $\log_{5.5}(\frac{1}{30.25}) = -2$

7. 3

8. $\log x = y$

9. -2

10. $$\begin{aligned} 5^x &= 25 \\ x &= 2 \end{aligned}$$

13. -2

14. $$\begin{aligned} 3^x &= \frac{1}{3^3} \\ x &= -3 \end{aligned}$$

17. -2

18.
$$\begin{aligned} 10^x &= 0.0001 \\ 10^x &= \frac{1}{10,000} \\ x &= -4 \end{aligned}$$

21. 10,000

22. $$\begin{aligned} &= 10^{\log(10^{0.5})} \\ &= 10^{0.5} \end{aligned}$$

23. 0.2

24. $$\begin{aligned} e^x &= 1 \\ x &= 0 \end{aligned}$$

27. $\ln(e^{\frac{1}{2}}) = \frac{1}{2}$

28.
$$\begin{aligned} e^x &= e^{\frac{5}{4}} \\ x &= \frac{5}{4} \end{aligned}$$

33. Between 2 and 3 because $5^2 = 25$ and $5^3 = 125$

34. 3 and 4 because 100 is between $\begin{matrix} 3^4 = 81 \\ 3^5 = 243 \end{matrix}$

43.

$$\log_5(625) = x$$
$$\frac{\log 625}{\log 5} = x$$
$$4 = x$$

44.

$$\log_2(0.25) = t$$
$$\frac{\log 0.25}{\log 2} = t$$
$$-2 = t$$

47.

$$\log_3 13 = x$$
$$\frac{\log 13}{\log 3} = x$$
$$2.33 \approx x$$

48.

$$\log_{15}(2) = x$$
$$\frac{\log 2}{\log 15} = x$$
$$0.256 \approx x$$

53.

$$97.2 = 0.06(4.92)^{\frac{x}{4}}$$
$$\frac{97.2}{0.06} = 4.92^{\frac{x}{4}}$$
$$1620 = 4.92^{\frac{x}{4}}$$
$$\log_{4.92}(1620) = \frac{x}{4}$$
$$\frac{\log(1620)}{\log(4.92)} = \frac{x}{4}$$
$$4.638 \approx \frac{x}{4}$$
$$18.55 \approx x$$

54.

$$\begin{aligned} \frac{100}{7.2} &= e^{2x+6} \\ 13.8 &= e^{2x+6} \\ \log_e 13.8 &= 2x+6 \\ \ln 13.8 &= 2x+6 \\ 2.62 &\approx 2x+6 \\ -3.38 &\approx 2x \\ -1.688 &\approx x \end{aligned}$$

55.

$$\begin{aligned} 164 &= 10^x \\ \log_{10} 164 &= x \\ \log 164 &= x \\ 2.21 &\approx x \end{aligned}$$

56.

$$\begin{aligned} 13^{5x} - 4 &= 39 \\ 13^{5x} &= 43 \\ \log_{13}(43) &= 5x \\ \frac{\log 43}{\log 13} &= 5x \\ 1.466 &\approx 5x \\ 7.33 &\approx x \end{aligned}$$

57.

$$\begin{aligned} B(d) &= 2.512^d \\ 5.445 &= 2.512^d \\ \log_{2.512}(5.445) &= d \\ \frac{\log(5.445)}{\log(2.512)} &= d \\ 1.839 &\approx d \end{aligned}$$

59. $F(16) = 440(2)^{\frac{16}{12}} = 1108.73$ The frequency is 1108.73 Hertz 16 keys to the right of Concert A. $F(-10) = 246.94$ The frequency is 246.94 Hertz 10 keys to the left of Concert A.

60.

$$\begin{aligned} 659.255 &= 440(2)^{\frac{n}{12}} \\ 1.498 &= 2^{\frac{n}{12}} \\ \frac{\log 1.498}{\log 2} &= \frac{n}{12} \\ 6.996 &\approx n \end{aligned}$$

About 7 keys to the right of Concert A the frequency is 659 Hertz.

65.

$$\begin{aligned} A(t) &= 500e^{-0.1386t} \\ 100 &= 500e^{-0.1386t} \\ \frac{100}{500} &= e^{-0.1386t} \\ \ln(0.2) &= -0.1386t \\ \frac{-1.609}{-0.1386} &= t \\ 11.612 &\approx t \end{aligned}$$

74.

$$\begin{aligned} 3^5 &= x \\ 243 &= x \end{aligned}$$

75.

$$\begin{aligned} 4^0 &= x \\ 1 &= x \end{aligned}$$

77.

$$\begin{aligned} e^{-5} &= x \\ \frac{1}{e^5} &= x \end{aligned}$$

78.

$$\begin{aligned} 6^3 &= 24x \\ 216 &= 24x \\ 9 &= x \end{aligned}$$

82.

$$\begin{aligned} \sqrt{\log_3(x)} &= 2 \\ \log_3(x) &= 4 \\ 3^4 &= x \\ 81 &= x \end{aligned}$$

83.

$$\ln x = (0.2)^2$$
$$\ln x = 0.04$$
$$e^{0.04} = x$$

88.

$$\log_x(25) = 2$$
$$x^2 = 25$$
$$x = 5$$

89.

$$x^{-3} = \frac{1}{64}$$
$$x^{-3} = 4^{-3}$$
$$x = 4$$

93.

$$10^1 = \log x$$
$$10 = \log x$$
$$10^{10} = x$$

94.

$$\ln(\ln x) = 2$$
$$e^2 = \ln x$$
$$e^{e^2} = x$$

96.

$$P(x) = -\log x$$
$$P(4.467 \cdot 10^{-8}) = -\log(4.467 \cdot 10^{-8}) = 7.35$$
$$P(3.548 \cdot 10^{-8}) = -\log(3.548 \cdot 10^{-8}) = 7.45$$

97. Battery Acid:

$$P(x) = -\log(x)$$
$$0.3 = -\log(x)$$
$$-0.3 = \log(x)$$
$$10^{-0.3} = x$$

Orange Juice:

$$P(x) = -\log(x)$$
$$4.3 = -\log(x)$$
$$-4.3 = \log(x)$$
$$10^{-4.3} = x$$

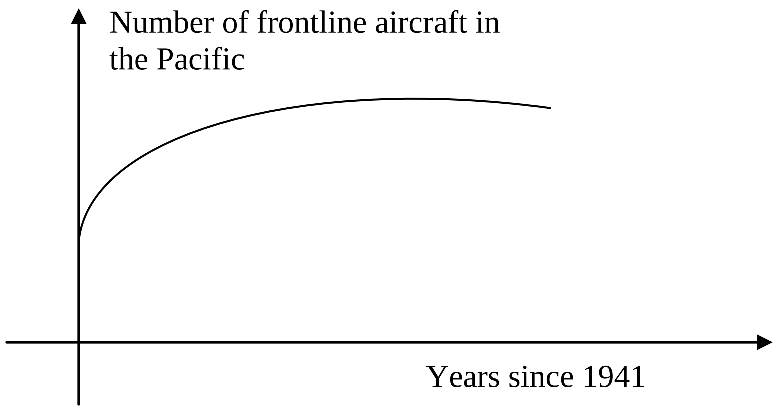
Sea water:

$$P(x) = -\log(x)$$
$$8 = -\log(x)$$
$$-8 = \log(x)$$
$$10^{-8} = x$$

Bleach:

$$P(x) = -\log(x)$$
$$12.6 = -\log(x)$$
$$-12.6 = \log(x)$$
$$10^{-12.3} = x$$

107.



The graph is increasing concave down. This means that as the years increase after 1941 the number of frontline aircraft in the Pacific continues to increase but at a decreasing rate.

108.

$$= 3444.18 + 12397.16(\ln 2.5)$$
$$\approx 14803$$

There was a total of 14,803 US front-line combat aircraft in 1944

109.

$$9000 = 3444.18 + 12397.16(\ln t)$$
$$5555.82 = 12397.16(\ln t)$$
$$0.448 = \ln t$$
$$e^{0.448} = t$$
$$1.56 \approx t$$

Halfway through 1942 there were 9,000 US front-line combat aircraft in the Pacific.